2017

20th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

Team Control Number: 8121

Problem Chosen: B: Ski Slope

Summary:

Wasatch Peaks Ranch is a prime location for a ski resort. Given the magnitude of the property and the randomness of nature, it is impossible to find a perfect ski resort layout. Our complete ski slope design is of Olympic caliber and will compete as one of the top ski resorts in all of North America.

Our model begins with extensive path generation which entails hundreds of trails being created, covering nearly 60 km. A subset of these trails were chosen based on their Trail Quality Index (TQI), a single value which determines the appropriateness of trails for skiers given their skill level, sharpness of curves, width of narrowest point, roughness, steepness, flatness, variability in directional derivatives, and points of kinetic energy loss. Using the data for the 40 km subset of trails, we simulated 120 km of trails using normal distributions for each of the aforementioned values. Our final design boasts more than 162 km of slopes with 38 beginner trails (20.4%), 74 intermediate trails (39.8%), and 74 expert trails (39.8%). Our beginner and intermediate trails have an mean TQI of 3.7 compared to a TQI of 2.4 for Whistler Blackcomb's premier ski trail. Any TQI values greater than 0.5 are considered acceptable.

The aggregate of all trails is our Wasatch Peaks Ranch ski resort design. To quantitatively compare our resort with others, we created a Resort Rating system which rates the resorts from 0.1 to 10 (10 being the best), based on a variety of factors, including ski lifts, trail density, vertical drop, and direction that the slope is facing, which is critical for snow quality. Using this system, Wasatch Peaks Ranch achieved a Resort Rating score of 7.44, ranking 3rd among 16 top North American ski resorts. To put this into more context, Park City Mountain, known as one of the best ski resorts, and previous Winter Olympic site, received a score of 8.67 using our rating and was ranked #1. Sugarloaf Mountain, ranked extremely poorly on ZRankings.com, received a 4.74. The system is proven reliable through its strong correlation with existing ranking systems and its vulnerability to sensitivity analysis.

Our rating systems are based on weighted composite scores in the various factors we incorporated, so that it truly reflects what skiers are looking for. It also incorporates logistic models to make sure that ratings below the minimum and ratings above the maximum cannot happen.

To measure Olympic compatibility and readiness, we used a similar system, which resulted in our Olympic readiness score of 7.8, well within the range of previous Olympic sites: Squaw Valley's 7.0 to Park City Mountain's 8.3.

Dear Ms. Mogul,

Wasatch Peaks Ranch is a prime location for a ski resort. Given the magnitude of the property and the randomness of nature, it is impossible to find a perfect ski resort layout. That being said, we are pleased to be able to present to you a complete design of Olympic caliber ski slopes that can compete with top ski resorts all around North America.

As stated previously, the size of Wasatch Peaks Ranch is much greater than that needed of a ski resort. Our slopes are located in two main clusters, one in the south among Francis Peak and Round Top and one in the north among Thurston Peak and Jacob's Peak. The remaining land can be utilized for commercial use or potential future Olympic village expansions.

Our design meets all requirements laid out in the request. The resort boasts more than 162 km of slopes. The distribution of trails is roughly 20% rated beginner, 40% rated intermediate, and 40% rated difficult with 38 beginner trails (20.4%), 74 intermediate trails (39.8%), and 74 expert trails (39.8%). Our design empowers skiers with trail length options ranging from a beginner's endurance to a whopping 3.38 kilometers for skiers looking to challenge themselves.

To rank our ski area among other existing resorts, we created a Resort Rating system. We included a multitude of factors, including the fit of the resort for beginners, intermediates, and experts, the quality of the snow, the quality of proposed lifts, the steepness of the mountain range, and crowdedness of the trails. Using this system, and weighting the different factors based on their value to the customer and potential skier, Wasatch Peaks Ranch achieved a rating of 7.44, on a scale from 0.1 to 10, where (10 is the best). To put this in perspective, the worst resort, Silver Star, had a rating of 4.47, and Vail (one of the best resorts according to ZRankings.com) had a rating of 7.12.

To measure Olympic compatibility and readiness, we used a similar system, except weighed the factors differently to make an Olympic readiness rating. When comparing our Olympic readiness rating of 7.8 to other sites previously used for the Winter Olympics, we were well within the range (Squaw Valley's 7.0 to Park City Mountain's 8.3). However, we should still try to improve the rating, as three other sites are also above that Olympic readiness score (Beaver Creek- 7.8 score, Big Sky Resort- 8.2 score, and Whistler Blackcomb- 8.2 score).

Unfortunately, we did not have the data around the actual quality of other factors, such as accommodations and living, safety of lifts, etc., which would help to make our score more accurate. To better compare our resort to theirs, it would viable to have data on more of the non-skiing aspects which are still inherently important to the overall ski trip.

Thank you for contacting us to develop the ski resort. We hope that you find the flexibility and sensitivity of our model useful and applicable to not just this resort, but other resorts should you choose to develop them. We are confident that presenting our design to the winter sports enthusiasts you represent is a worthwhile endeavour, and that the optimization of our trails can help you in future developments.

Thank you again for letting us create this model for you,

We look forward to presenting a more refined model at a later date inform your future purchasing decisions.

Team 8121

1 - Introduction

1.1 Background

First Tracks Online Ski Magazine [1] describes Wasatch Peaks Ranch as what may be America's last great ski resort site. The almost 13,000 acre Utah ranch has great potential in becoming the host of a future Winter Olympic, boasting a 4750 foot drop among its 24 peaks. Ms. Mogul, an agent of a wealthy group of winter sports fans, has tasked us with identifying potential ski slopes and trails on the property in order to develop Wasatch Peaks Ranch into one of the top ski resorts in North America and a potential future Winter Olympics location. Ms. Mogul provided us with a brochure for Wasatch Peaks Ranch, a topographic map, and a partial list of North American ski resorts.

1.2 Restatement of Problem

Given the subjective nature of an ideal ski area design, our team narrowed the constraints to a list of essential features needed in a Winter Olympic caliber resort layout. Our design would have to provide at least 160 km of trails, match a distribution of roughly 20% rated beginner, 40% rated intermediate, and 40% rated difficult slopes, and be Winter Olympics compatible. The solution would be presented as a series of trails with ski lifts. The ski area would then need to be ranked among existing North American ski resorts. The credibility of our ski area design can be gauged by its rank compared to other top North American ski resorts. The credibility of our ranking system can be measured by its similarity to existing rankings and its immunity to sensitivity analysis. A brief report of our findings would then be written for Ms. Mogul, which she would present to the group she is representing.

2 - Definitions, Assumptions, Variables

2.1 Definitions

Terms used throughout the paper are defined here.

Slope: The snow covered downward skiable area in the resort

Trail: The designated recommended paths upon which trail length and difficult levels are measured

2.2 Assumptions

The following assumptions were necessary in our model to ensure maximum practicality and usability.

- Our resort would not be responsible for all Winter Olympic events, but only one, such as alpine skiing. As a parallel, the 2018 Pyeongchang Winter Olympics will have Yongpyong Resort and Jeongseon Alpine Centre as the only hosts to the alpine skiing events [2].
- Ski trails that merge are measured as separate trails in the total trail length calculations.
- Existing infrastructure (roads) and problematic environmental features (rocks) do not pose a significant obstacle and can be removed.
- Skier slope preferences are based on skill levels:
    - Beginners prefer wider slopes with smooth curves, smooth terrain, and a gradual slope (results in less velocity). Beginners ski mainly on green circle difficulty trails.
    - Intermediates prefer slightly narrower slopes with sharper curves,, rougher terrain, and a steeper slope (results in greater velocity) when compared to beginners. Intermediate ski mainly on blue square difficulty trails.
    - Experts prefer significantly narrower slopes with sharper curves, rougher terrain, and a steeper slope (resulting in even greater velocity than intermediates). Experts ski mainly on black diamond difficulty trails.

- Olympic alpine skiing will occur primarily on our expert black diamond slopes, but will still use some intermediate slopes, and rarely beginner ones (possibly for warm up).
- Slopes facing in a direction between North and East will have improved snow quality over slopes facing the West. Southward facing slopes will have decreased snow quality.

3 - Model Description

Our model consists of two parts, a Wasatch Peaks Ranch trail optimization and a North American resort ranking. The parts are independent, however we utilize the data obtained from the optimized trail model to compare the hypothetical Wasatch Peaks Ranch to existing resorts.

3.1 Finding Optimal Trails

To find optimal trails, we created a Trail Quality Index (TQI) to gauge quality of each trail. Big data was collected on a large collection of Google Earth Pro generated trails. This data was narrowed down using the TQI results and requirement for varying lengths and difficulty degree.

3.1.1 Variables

The following variables are used within our model:

D : skill level constant (no unit, see below)

s : sharpness of curves on a trail (% of trail)

w : narrowest point on a trail (meters)

r : roughness of trail (ratio)

$\frac{\Delta z}{l}$: steepness of trail (meters / meter)

f : flatness of trail (ratio)

IQR : interquartile range of slopes at each point on the trail (meters /meter)

E : energy loss (% of trail)

3.1.2 Difficulty Level Constant

Difficulty Rankings:

Green Circle	Beginner Slope	D = -1
Blue Square	Intermediate Slope	D = 0.9
Black Diamond	Expert Slope	D = 1

The difficulty levels were assigned to skill levels to reflect the Trail Quality Index described below. A negative D value will decrease the TQI of difficult trails, which are undesirable to beginners. A high positive D value will produce the opposite effect, raising the TQI for difficult trails.

3.1.3 Trail Quality Index (TQI)

The TQI measures the quality of a trail based on its difficulty level. The simplest factor that would affect how well a trail is designed is its average slope. A high directional derivative is desirable by expert skiers while a low average directional derivative is desirable for beginners. Our TQI function is directly proportional to the average directional derivative for experts and inversely proportional for beginners. This can be expressed as:

$$TQI = \left(\frac{\Delta z}{l}\right)^D$$

where Δz is the change in elevation over the length of the trail and l is the total length of the trail.

After accounting for average slope, our model addresses more complex factors, such as roughness of terrain, r, and sharpness of the trail, s. We measured the percent of the entire trail that passes through rocky or tree covered terrain. The sharpness of the trail is measured as the percent of the trail that comprises of a turn that has degree of curvature less than 90°, an acute angle. Expert skiers prefer rough terrain and increased sharpness while beginners prefer the opposite. Since both these parameters can only have values from 1-100, they can only add to the TQI of a trail, not decrease it, so we used the ratio between the parameter value and the average value for both of those parameters. TQI relates to r and s in the same way it relates to $\frac{\Delta z}{l}$. Appending r and s to TQI results in:

$$TQI = (\tfrac{\Delta z}{l})^D \times r^D \times s^D$$

Another factor that affects TQI is the width of the trail, w. We decided to compute the narrowest part of the trail. However, unlike the previous variables that affect TQI, w is inversely proportional for expert skiers and directly proportional for beginners, meaning beginners want wider trails, while experts prefer more narrow trails. TQI now takes the following form:

$$TQI = \frac{(\frac{\Delta z}{l})^D \times r^D \times s^D}{w^D}$$

Unlike previous inputs to TQI, which are conditional to skier skill level, percent of the trail that is flat (f) and the energy loss of uphill travel (E) will be inversely proportional to TQI for all expertise levels. Energy loss was determined by evaluating the percentage of the trail that is going uphill. Since both these parameters can only have values from 1-100, they can only decrease the TQI of a trail, not add to it, so we used the ratio between a trail's value and the average value for both of those parameters to ensure that the parameters can affect the Trail Quality Index positively or negatively.

$$TQI = \frac{(\frac{\Delta z}{l})^D \times r^D \times s^D}{w^D \times f \times E}$$

The final factor determined to affect the Trail Quality Index was the variability of its directional derivatives in the direction of the path. This value is essentially the mathematical slope of the trail at a given point on the trail, however this nomenclature is not used to avoid confusion. Instantaneous change in elevation at each point on the trail will be referred to by its technical term, the directional derivative in the direction of the trail. We used IQR over range as a measure of variability in this case due to the interquartile ranges' resistance to outliers. We concluded that more variability in the directional derivatives would be detrimental to the Trail Quality Index since the difficulty level does not necessarily apply to the entire trail. However, Trail Quality Index should be positively affected if the directional derivatives do not vary much (low IQR). As a result, TQI and IQR are inversely proportional. Additionally, we concluded that variance in directional derivatives is not as important as the other factors in TQI since a relatively flat trail with one steep drop would still be a test of skill. At its current stage, Trail Quality Index model looks like:

$$TQI = \frac{(\frac{\Delta z}{l})^D \times r^D \times s^D}{w^D \times f \times E \times IQR^{1/2}}$$

However, with the current scaling, our values ranged from 10^{-4} to 10^6, so we multiplied the TQI by 10^2 and then logarithmized the TQI to help conceptualize the scores: a number between -10 and 10 is easier to comprehend than values that are orders of magnitude different. Therefore, our true final model for the Trail Quality Index is:

$$TQI = log(10^2 \times \frac{(\frac{\Delta z}{l})^D \times r^D \times s^D}{w^D \times f \times E \times IQR^{1/2}})$$

3.1.4 Path Generation

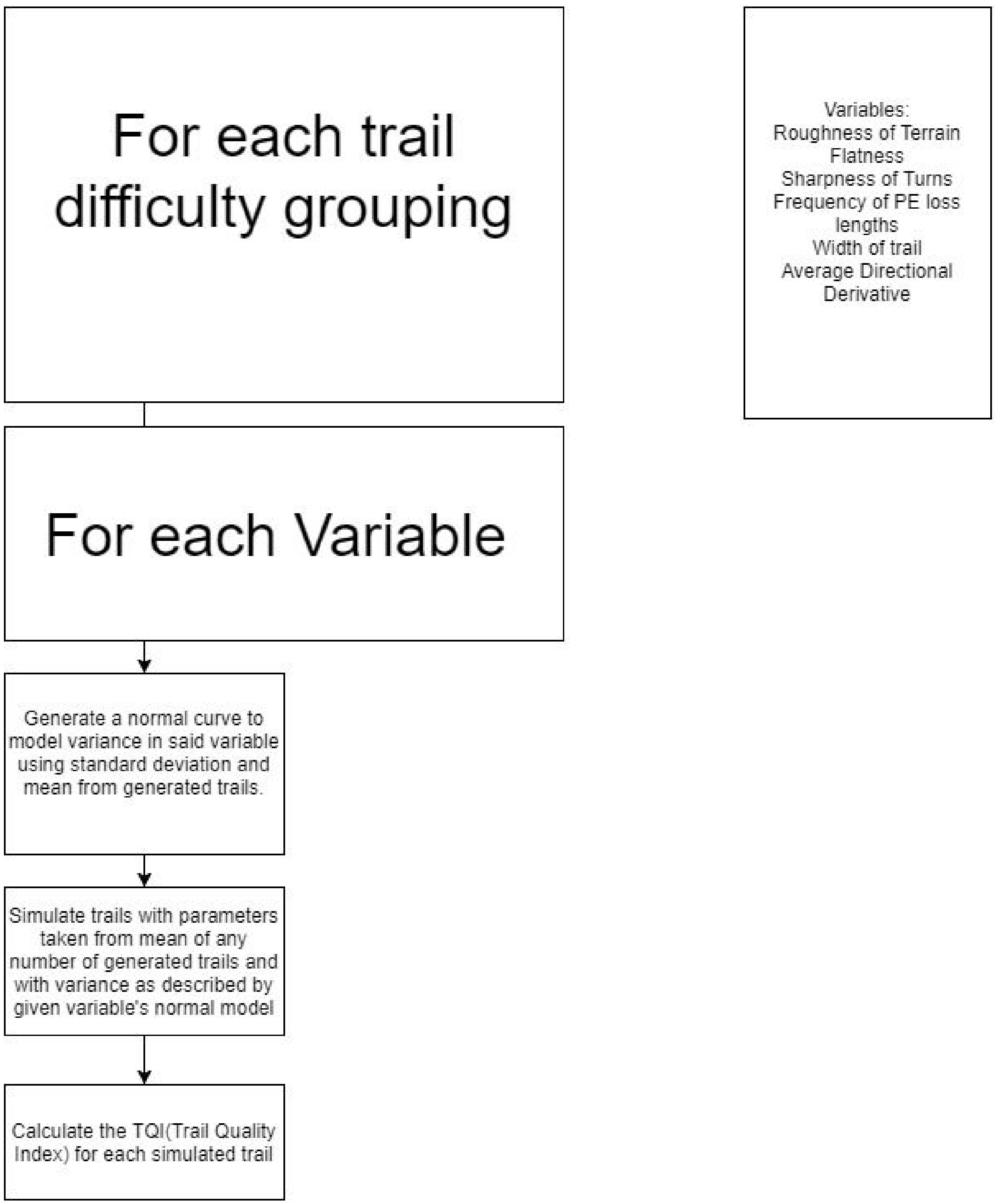
All the data required for this model can be obtained through the Google Maps API in combination with Google Earth Pro [6]. We generated 60 km of trails using the factors defined in the previous section, and eliminated approximately 20 km of trails based on the TQI values. The following table gives data for a fall line -- the absolute shortest path down the mountain -- one very good trail from a different Ski resort, and one of our generated paths that returned a reasonably high TQI.

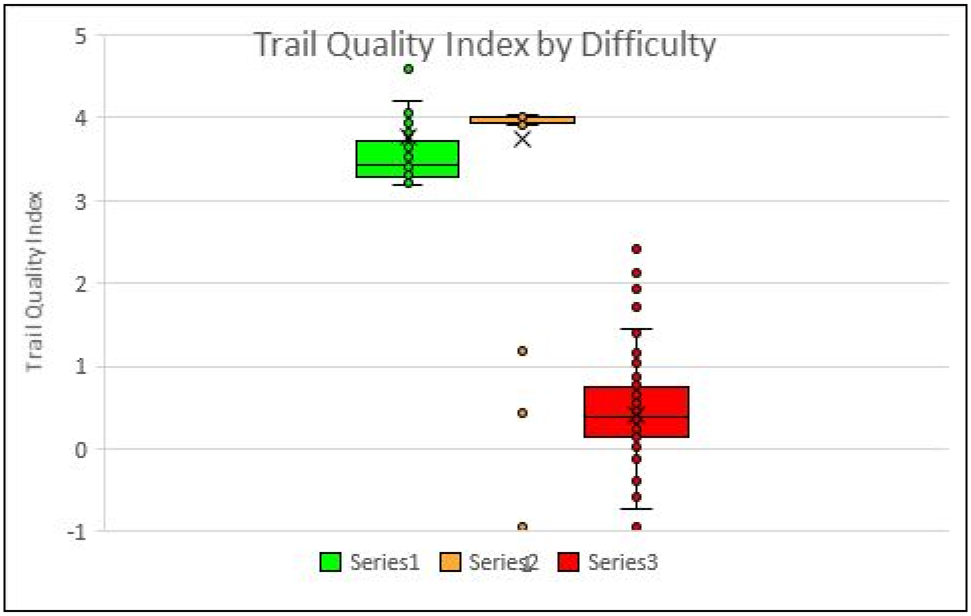
Name	Fall Line	Generated Slope	Whistler Blackcomb (Canada)
$\frac{\Delta z}{l}$	0.2620394199	0.6465136522	0.2545933426
r	0.03123886039	1.09456007	1.249554416
s	0.05320682122	2.770024777	1.276648876
w	15	15	6.36
E	1.555555556	1	1
f	0.3428007942	0.1580578512	0.0554228829
IQR	0.1975302519	0.3787962148	0.1926055728
TQI	-1.911806289	2.128190807	2.419193031

This information provides context to the range of solutions we can get from this model: values less than 0 should be eliminated and positive values above 2 are very good trails.

Based on the distribution of data for the 40 km of trails that we generated on the map, we simulated 100 km of remaining trails. We used the standard deviations for the directional derivative at each point and random gaussian functions to simulate the changing altitude

Simulating Trails





Slopes were optimized to have varying length and the correct proportion of beginner, intermediate, and expert trails.

3.2 Developing a Resort Ranking System

We developed a resort ranking system that assigns ratings based on popular characteristics. The ranking system assigns a value 0.1 to 10 to each resort, with 10 being the best. The ranking is obtained through a weighted average of different factors which we assume affect the overall rating of the resort.

3.2.1 Inputs

* Note: The following ratings are assigned to whole resorts, not individual slopes.

Q: Snow quality rating

w: Average annual snowfall per year in meters

d: Difference in proportion of slopes facing north/east direction and those facing south

L: Ski lift rating

P: Trail density rating

F_{skill}: Fit of resort to specified skill level (i.e. b=beginner, i=intermediate, and e=expert)

n: Total number of slopes for the specified skill level

t : Total length of ski trails for the specified skill level

S : Steepness rating

v : Vertical drop from max elevation to minimum elevation in meters

l : Length of all ski trails in the resort

U: Resort Rating

U_{skill} :Resort Rating specific to desirability for given skill level

3.2.2 Weighting Inputs

To better understand the effectiveness of our resort design and to be able to quantitatively compare it with other resorts, we incorporated a variety of factors into the "resort rating." Various skiers prefer different features in their slopes and trails; our rating system weights the skier preferences based on the ratio provided by Ms. Mogul in the problem statement (20% beginner, 40% intermediate, 40% expert).

$$U = 0.2U_b + 0.4U_i + 0.4U_e$$

U is our "resort rating," and U_b, U_i, and U_e are the desirability of the resort for skiers of given skill levels. In general, U_{skill} depends on the snow quality, ski lifts, trail density, and fit rating for each respective skill level. To model this, we made U_{skill} a weighted average, with weights assigned based on the importance of each factor to skier desirability.

Factor	B-Weight (coefficients)
Snow Quality (Q)	2.5
Ski Lift Rating (C)	1.5
Trail Density (P)	2
Fit Rating (F_{skill})	4
Total Weight	10

The weighting is integrated to create our generalized U_{skill} rating equation:

$$U_{skill} = \frac{(2.5Q + 1.5L + 2C + 4F_{skill})}{10} = 0.25Q + 0.15L + 0.2P + 0.4F_{skill}$$

After inputting U_{skill} back into the U formula for each skill level (b=beginner, i=intermediate, and e=expert) and simplifying values, we get:

$$U = 0.25Q + 0.15L + 0.2P + 0.08F_b + 0.16F_i + 0.16F_e$$

3.2.3 Deriving Inputs

3.2.3.1 Deriving Snow Quality Rating *Q*:

The snow quality rating Q is obtained through a logistic curve, which exhibits two asymptotes limiting the rating to a range from 0.1 (*min*) and 10 (*max*). A generic formula for quality of snow, Q, as a function of the factors that influence snow quality, Q_{change} can be modeled as:

$$Q = \frac{max \times min}{min + (max - min) \times e^{-max \times k_Q \times (Q_{change})}} = \frac{1}{0.1 + 9.9e^{-10 \times k_Q \times Q_{change}}}$$

where k_Q is an arbitrary positive constant used to adjust the sensitivity of snow quality to Q_{change}.

The factors influencing snow quality, Q_{change}, can be modeled based on the amount of natural snowfall and the direction the snow is facing. Snow facing south is significantly inferior to that facing north or east [5], while snow facing west has no significant advantage or disadvantage. If Q_{change} increases proportionally to snowfall and the difference between the proportion of slopes facing north/east and south, then:

$$Q_{change} = w \times d$$

where w is annual average snowfall in meters and d the difference in proportion of slopes facing north/east and those facing south. Through adjusting of k and analyzing its impact on snow quality Q, we have set $k = 0.1$. This provides a significant variation between different resorts snow quality. The final snow quality rating Q can be modeled as:

$$Q = \frac{1}{0.1 + 9.9e^{-wd}}$$

3.2.3.2 Deriving Fit Rating F_{skill}:

Similar to the snow quality rating, the fit rating, F_{skill}, for each different skill level is logistic in nature, ensuring a maximum rating of 10 and minimum of 0.1. F_{skill} can be obtained through:

$$F_{skill} = \frac{1}{0.1 + 9.9e^{-10 \times k_{skill} \times F_{change}}}$$

where k_{skill} is an arbitrary positive constant allowing us to adjust the sensitivity of fit rating to F_{change} We set k_{skill} values such that the "average" resort would have have a rating of 5 in F_{skill} for all three skill levels.

Skill Level:	k_{skill} value
Beginner	$\frac{3}{500}$
Intermediate	$\frac{1}{5500}$
Expert	$\frac{1}{26400}$

The factors influencing rating, F_{change}, can be modeled as a function of the number of slopes and the total length of trails for that specific skill level. And if F_{change} is directly proportional to those two factors, then

$$F_{change} = n \times t$$

where n is the number of slopes designated at that specified skill level and t the length of the trails for the specified skill level. Beginners prefer less steepness than intermediates, who prefer less than experts. For beginners, F_{change} is inversely proportional to steepness modeled as:

$$F_{change\ (b)} = \frac{n \times t}{S(v,\ l)}$$

where $S(v, l)$ is the steepness rating as a function of v, the vertical drop in meters, and l, the length of all trails in meters. Since intermediates want more steepness than beginners, but less than experts, F_{change} does not change and would remain as:

$$F_{change\ (i)} = n \times t$$

For experts, F_{change} is proportional to the steepness, as they are looking for greater thrill:

$$F_{change\ (e)} = n \times t \times S(v,\ l)$$

To calculate steepness rating, we set the steepness inversely proportional to the length of all the trails (which would reflect the length of the mountain range). Steepness rating would then be:

$$S(v, l) = k_S \frac{v}{l}$$

where k_S is used to adjust the ratio so that the resort with the greatest drop over the shortest distance gets a steepness rating of approximately 10. With $k_S = 10/11$ (as the greatest ratio was ~11), steepness rating would be:

$$S(v, l) = \frac{10v}{11l}$$

For different skill levels, the fit rating, F_{skill} is

$$F_{skill} = \frac{1}{0.1 + 9.9e^{-10 \times k_{skill} \times F_{change}}}$$

Consequently, a table of F_{skill} values can be made:

Skill Level	k_{skill}	F_{change}
Beginner	$\frac{3}{500}$	$n \times t \times \frac{11l}{10v}$
Intermediate	$\frac{1}{5500}$	$n \times t$
Expert	$\frac{1}{26400}$	$n \times t \times \frac{10v}{11l}$

3.2.3.3 Deriving Ski Lift Rating *L*:

The lift rating *L* is the ratio between the number of lifts in the resort and the number of slopes (lifts per slope). Therefore, we derive:

$$L = 25 \times \frac{number\ of\ lifts}{number\ of\ slopes}$$

Where 25 is to adjust the ratio so that the average of 0.2 lifts/slope gives a lift rating *L* of approximately 5.

3.2.3.4 Deriving Trail Density Rating *P*:

The trail density rating, *P*, can be obtained in a similar fashion. We used the ratio of total length of trails (in meters) to total size of skiable area (in hectares) to find the kilometers of ski trail per hectare:

$$ratio = \frac{length\ of\ trails}{size\ of\ skiable\ area}$$

However, since a larger ratio of trail length to skiable area means more crowdedness, we want to find the "uncrowdedness" of trails:

$$P = 1 - \frac{\textit{length of trails}}{\textit{size of skiable area}}$$

Since that model of trail density rating gives values less than or equal to 1, we add a scaling factor of 10, so that when there is absolutely no crowdedness (no trail in any given hectares of skiable area), $P = 10$. The final formula for the trail density rating is

$$P = 10 \times \left(1 - \frac{\textit{length of trails}}{\textit{size of skiable area}}\right)$$

3.2.4 Overall Resort Rating

The overall resort rating can be obtained by computing the weighted average of all skill level ratings. The ratio of 20% beginner, 40% intermediate, and 40% expert is the suggested distribution of slopes given to us by Ms. Mogul. Thereupon, we created our final rating equation:

$$U = 0.2U_b + 0.4U_i + 0.4U_e$$

Adding in the equations of U_{skill} as a function of snow quality rating Q, ski lift rating L, trail density rating P, and fit rating for the different levels F_{skill} (b=beginner, i=intermediate, and e=expert), we get the following model:

$$U = 0.25Q + 0.15L + 0.2P + 0.08F_b + 0.16F_i + 0.16F_e$$

where:

$$Q = \frac{1}{0.1 + 9.9e^{-wd}} \ ,$$

$$L = 25 \times \frac{\textit{number of lifts}}{\textit{number of slopes}} \ ,$$

$$P = 10 \times (1 - \frac{\textit{length of all trails}}{\textit{size of skiable area}}),$$

and,

$$F_{skill} = \frac{1}{0.1 + 9.9e^{-10 \times k_{skill} \times F_{change}}}$$

Skill Level	k_{skill}	F_{change}
F_b	$\frac{3}{500}$	$n \times t \times \frac{11l}{10v}$
F_i	$\frac{1}{5500}$	$n \times t$
F_e	$\frac{1}{26400}$	$n \times t \times \frac{10v}{11l}$

For quick reference:

Q: Snow quality rating

w: Average annual snowfall per year in meters

d: Difference in proportion of slopes facing north/east direction and those facing south

L: Ski lift rating

P: Trail density rating

F_{skill}: Fit of resort to specified skill level (i.e. *b*=beginner, *i*=intermediate, and *e*=expert)

n: Total number of slopes for the specified skill level

t: Total length of ski trails for the specified skill level

v: Vertical drop from max elevation to minimum elevation in meters

l: Length of all ski trails in the resort

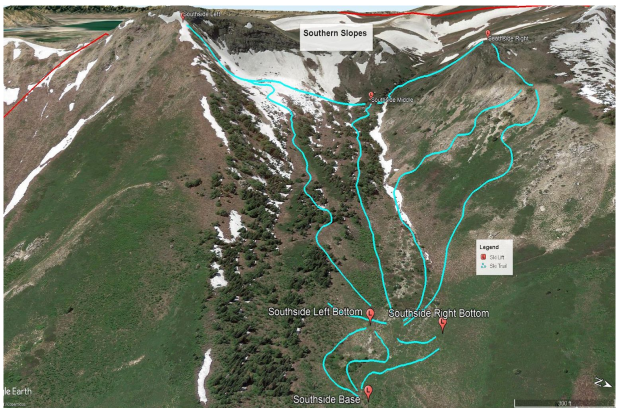
U: Resort Rating

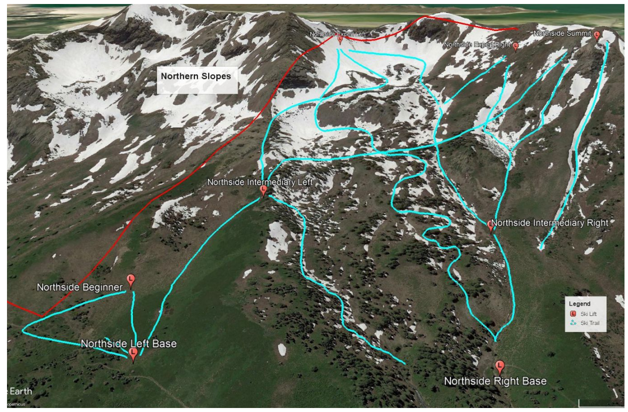
U_{skill}: Resort Rating specific to desirability for given skill level

4 - Analysis

4.1 Obtaining Possible Trails

Optimal trails are located along the Francis Peak and Round Top and the Thurston Peak and Jacob's Peak.





4.1.1 Obtaining Ski-ability Index Numbers

The trails created above were simplified using the Trail Quality Index described in the model description. Here are three trails, one of each skill level, that were rated on the scale.

4.1.2 Finalizing Trails

We finalized our slope design by choosing only trails that scored a 3 or above on the TQI. The following slopes were chosen.

4.1.3 Ski Lifts

In addition to the final trails, we included locations for ski lifts. This feature is essential in any ski resort and should always be planned for by the trail designers.

4.2 Ranking

4.2.1 Ranking North American Resorts

Using the process described in **3.2 Developing a Resort Ranking System** and data from [3] and [4], 16 prominent North American ski resorts scored the following ratings:

Resort	Rating (0.1-10)	Resort	Rating (0.1-10)
Park City Mountain	8.67	Jackson Hole	6.25
Whistler Blackomb	8.46	Killington	5.96
Big Sky Resort	7.96	Steamboat Springs	5.62
Beaver Creek	7.64	Lake Louise	5.03
Squaw Valley	7.14	Sun Peaks	4.88
Vail	7.12	Sugarloaf Mountain	4.74
Breckenridge (USA)	6.87	Silver Star	4.47
Winter Park Resort	6.69		

4.2.2 Ranking Wasatch Peaks Ranch

The hypothetical ski resort we created on Wasatch Peaks Ranch receives the following rating. This is the third highest rating in our North American ranking:

Resort	Rating (0.1-10)
Wasatch Peaks Ranch	7.44

4.2.3 Olympic Readiness

To create a measure of Olympic readiness, we used the same model, except weighted the individual U_{skills} different from our previous model, due to our assumption that for the

Olympics, expert level slopes are more important than intermediate level slopes which are more important than beginner level ones. The Olympic readiness score would be:

$$U = \frac{1}{6}U_b + \frac{2}{6}U_i + \frac{3}{6}U_e$$

According to this model, Wasatch Peaks Ranch has an Olympic readiness score of 7.8, on a scale from 0.1 to 10 (10 being most ready). Two other ski resorts have been used for the Olympics before: Squaw Valley and Park City Mountain. We are well above the baseline of Squaw Valley's Olympic readiness rating of 7.0(the lower of the two. However, there are three other resorts which also have an Olympic readiness rating above the baseline:

Resort	**Rating (0.1-10)**
Squaw Valley (Olympic)	7.0
Beaver Creek	7.8
Wasatch Peaks Ranch	**7.8**
Big Sky Resort	8.2
Whistler Blackomb	8.2
Park City Mountain (Olympic)	8.3

5 - Results and Conclusion

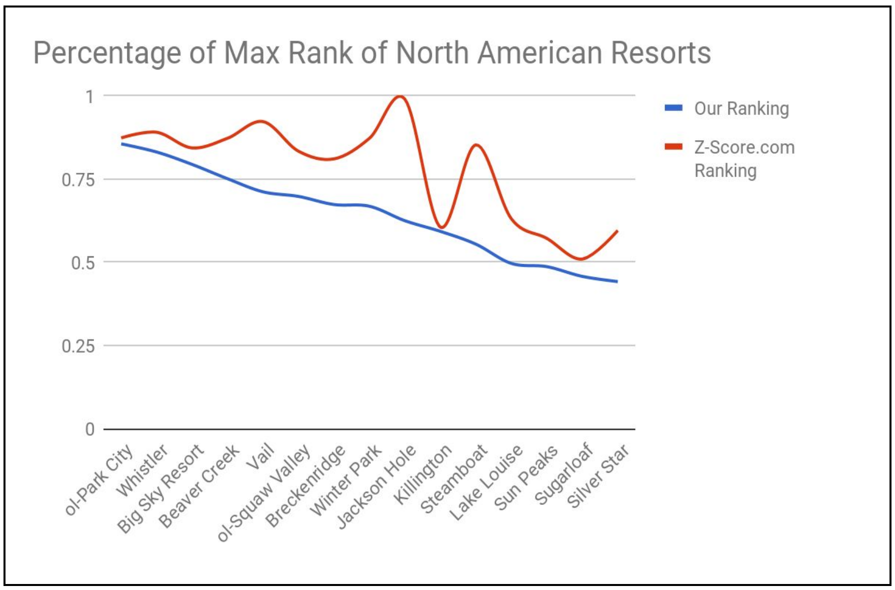
5.1 Addressing the Slopes

Our finalized trail design is appropriate because it meets the criteria laid out in **1.2 Restatement of Problem.** Our design boasts more than 162 km of slopes.. The distribution of trails is roughly 20% rated beginner, 40% rated intermediate, and 40% rated difficult with 38 beginner trails (20.4%), 74 intermediate trails (39.8%), and 74 expert trails (39.8%). Our design exhibits great variation in trail lengths, with a standard deviation of 657.45 meters. The design is

Olympic compatible as it ranks similarly as previous Olympics sites when we weight the courses to Olympic standard. Our resort is Olympic ready, with a score of 7.8, which is greater than the score of 7.0 from previous Olympic site Squaw Valley.

5.2 Comparison to ZRanking

ZRankings.com uses the PAF algorithm, which considers over 20 categories, to determine what ski resorts are the best [3]. The categories in their system are similar to those we employed, therefore comparing both ranking systems is justified. When graphing ZRanking's rank of the top North American resorts in the chart above in the ranking order we derived, we get a strong correlation. This correlation in the rankings reinforces the strength of both systems.



This chart shows the correlation of between our ranking system and that used by ZRankings.com

5.3 Ranking Sensitivity Analysis

Our ranking system, like all models, is not perfect. Our confidence in the model is high, however, as sensitivity analysis shows that small changes do not significantly affect the rankings.

For example, snow quality is measured as:

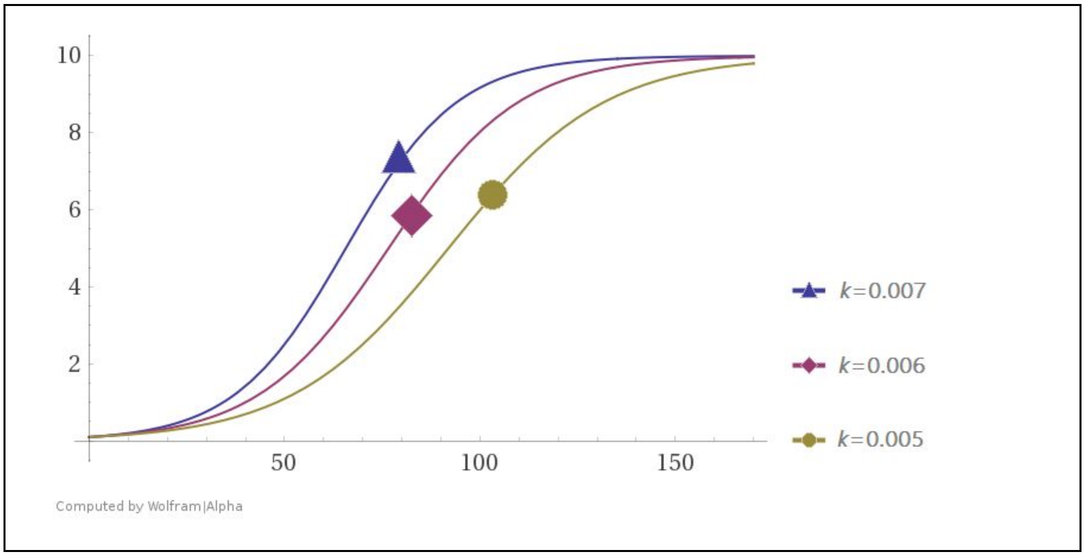
	Annual Average Snowfall in meters (w)	Difference in Proportion Facing N/E and Proportion Facing S (d)	Snow Quality Rating (Q)
Actual Wasatch Values	10.20	66.7%	9.0
Slightly increasing *d* (increasing N/E facing, decreasing S facing)	10.20	**73.3%**	9.5
Maxing *d*	10.20	**100.0%**	10.0
Halving *d*	10.20	**33.3%**	2.3
Slightly decreasing *w*	**9.5**	66.7%	8.5
Doubling *w*	**20.00**	66.7%	10.0
Halving *w*	**5**	66.7%	2.2

Changing average annual snowfall w creates large changes but only when there are drastic changes; otherwise, the snow quality rating changes appropriately to reflect changing snowfall. Similarly, changing the d also can have significant change (where halving d will decrease rating by more than half, as such a large decrease puts the difference at well below the average d of 51.6%).

Similarly, the fit rating F_{skill} also varies as the different inputs of the number of slopes for that skill level n, length of trails for that specific skill level (km) t, total length of trails (km) l, and vertical drop (m) v change. For F_b (fit rating for beginners), the rating increases proportionally to t and l and inversely proportionally to v:

$$F_b = \frac{1}{0.1 + 9.9e^{-10\times k_b \times n \times t \times \frac{11l}{10v}}}$$

As k_b is our adjusting constant so that we can change the sensitivity of our model to n, t, l, and v.



The chart shows the variation of F_b while k_b varies.

The above chart shows the variation of F_b for different values of k_b. We set $k_b = 0.006$ in our calculations as for the data set we used, $k_b = 0.006$ gave a wide range of reasonable F_b ratings. Using the same optimization strategy (of getting the largest range of fit ratings for a given skill level), we had $k_i = \frac{1}{5500}$ and $k_e = \frac{1}{26400}$.

5.4 Conclusion

Our model is appropriate because it ranks 3rd in our ranking system among top North American resorts. Our design meets and exceeds all requirements and our ranking is immune to sensitivity analysis. We can confidently write a letter to Ms. Mogul sharing our results.

6 - Works Cited

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